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LIFTING-SURFACE THEORY APPLIED TO ISOLATED RING WINGS AT ANGLE OF ATTACK

by

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ABSTRACT. Developed in this report are expressions for the lift, pitching moment, and location of the aerodynamic center due to free-stream angle of attack for an isolated cylindrical wing based on a solution of Weissinger's general boundary condition for ring wings. From the components of ring vorticity expanded in a Birnbaum series and satisfying the velocity boundary-flow condition, and from the Kutta-Joukowski relation for the force on a vortex surface, the lift, moment, and aerodynamic center can be expressed in terms of the Birnbaum coefficients. It was found that the lift and pitching moments for cylindrical ring wings are substantially different from the results for two-dimensional flat plate (thin) airfoils. Calculations show a significant shift in the aerodynamic center upstream of the 1/4-chord point for cylindrical ring wings.

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AN ACTIVITY OF THE BUREAU OF NAVAL WEAPONS

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FOREWORD

This report presents a method for determining the loadings on an isolated ring wing at angle of attack. The results of this method, based on linearized lifting-surface theory, were applied in estimating the pitching moments on a ring-wing tail configuration used as a control surface.

The work was conducted at the U. S. Naval Ordnance Test Station from October 1962 to April 1963 under Bureau of Weapons Task Assignment RUTO-3E-000/216-1F008-06-02.

The considered opinions of the Propulsion Division are represented in this report.

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CONTENTS

NEGATIVE NUMBERS OF ILLUSTRATIONS

Fig. 1 to 6, LHL-P 24660

NOMENCLATURE

$a_{\rho n}$	Fourier coefficient of $\alpha_n(\zeta)$
$C_{\nu n}$	Birnbaum coefficient of $g_n(\zeta)$
$C_{\nu 1}$	Birnbaum coefficient of $g_1(\zeta)$
$C_{\nu 1}^{(0)}$	Birnbaum coefficient for unit angle of attack
C_L	Total normal coefficient of lift
C_M	Total pitching moment coefficient
$g_n(\zeta)$	Fourier coefficient of $\gamma(\zeta, \phi)$
$g_0(\zeta)$	Ring vortex density due to conicity
$g_0^{*}(\zeta)$	Ring vortex density resulting from cambered section
$g_1(\zeta)$	Ring vortex density resulting from angle of attack
L	Length of cylindrical wing
M	Total pitching moment
R	Radius of cylindrical wing
$U_n(\eta)$	Velocity influence function
$U_1(\eta)$	Velocity influence function for α_1
V	Free-stream velocity
V_r	Free-stream radial component of velocity
x_{ac}	Aerodynamic center measured from mid-chord point
X, R, ϕ	Coordinates of cylindrical wing
XZ	Pitch plane
Y	Total net force normal to wing axis

$\alpha(\zeta, \phi)$	Local angle of incidence
$\alpha_n(\zeta)$, $n = 0, 1, 2 \ldots$	Fourier coefficient of $\alpha(\zeta, \phi)$
$\alpha_0^*(\zeta)$	Slope of wing profile for cambered section
α_0	Half-cone angle (conicity) of wing section
α_1	Free stream angle of attack
$\gamma(\zeta, \phi)$	Total ring vortex density
Δp	Force per unit area
$\zeta = 2X/L$	Dimensionless axial coordinate $-1 \leq \zeta \leq 1$
ζ'	Dimensionless variable of integration $-1 \leq \zeta' \leq 1$
$\eta = \lambda(\zeta - \zeta')$	Argument of velocity influence function
θ	Angular coordinate in Birnbaum expansion
$\lambda = L/2R$	Wing length-diameter ratio
ρ	Fluid density

INTRODUCTION

Two-dimensional airfoil theory is known to be in a highly developed state and to provide an accurate description of airfoil flow characteristics. In ordinary two-dimensional thin airfoil theory, two classes of problems exist: (1) the direct problem in which the velocity and pressure fields are known and the airfoil geometry is to be determined; and (2) the indirect problem in which the geometry is given and the flow field characteristics about the airfoil are to be determined. A typical problem in which the indirect method is employed is the calculation of spanwise loadings on finite wings for which the geometry of the wing is known.

Until recently, three-dimensional ring airfoil theory had been less well developed. However, techniques employing lifting-surface theory now account for three-dimensional effects due to curvature of the chord plane. It has been shown that, if the radius of curvature of the chord plane for a ring wing is small relative to the chord length, the aerodynamic characteristics for a ring wing differ markedly from the flow characteristics of a two-dimensional airfoil.

This report is concerned with loadings on the isolated cylindrical ring wing at angle of attack. A subsequent report will discuss loadings on ring wings of arbitrary profile with wing-body interference effects.

GENERAL FORM OF BOUNDARY CONDITION FOR RING WINGS

Using cylindrical coordinates X, R, ϕ with the origin at the wing center as represented in Fig. 1, a thin ring wing is replaced by a cylindrical vortex surface of radius R. This is equivalent to a thin airfoil-type approximation in which the velocities and pressures on the ring wing are represented by the velocities and pressures induced by a cylindrical vortex surface at the radial distance R, where R is interpreted to mean the average radial distance of the wing camber line. The net loading or difference in pressure per unit length per unit span induced by the vortex surface is represented by the Kutta-Joukowski relation

$$\Delta p = \rho V \gamma(\zeta, \phi), \qquad \zeta = \frac{2X}{L}, \qquad -1 < \zeta < 1 \tag{1}$$

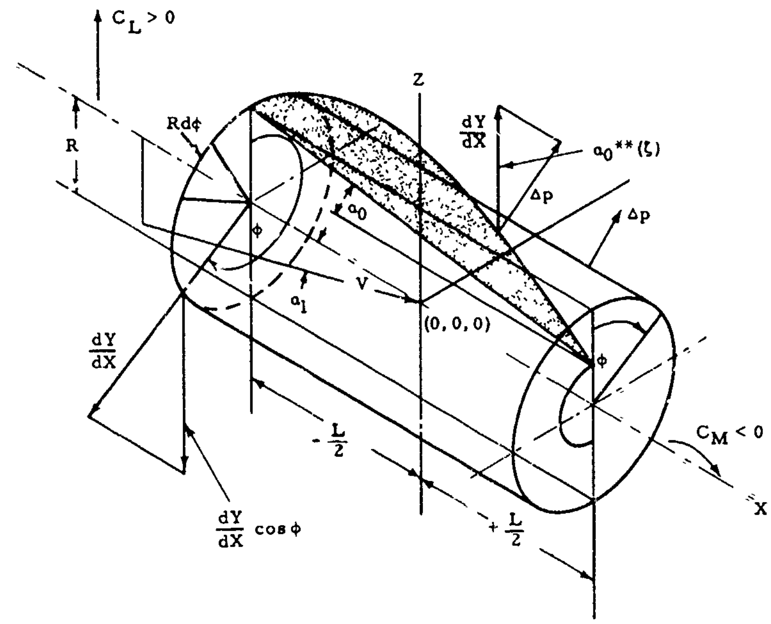

FIG. 1. Coordinates.

where $\gamma(\zeta, \phi)$ is the density of the ring vortices on the cylindrical surface. The density can be written

$$\gamma(\zeta, \phi) = V \sum_{n=0} g_n(\zeta) \cos n\phi \tag{2}$$

$$g_n(\zeta) = C_{0n} \operatorname{ctg} \frac{\theta}{2} + \sum_{\nu=1} C_{\nu n} \sin \nu\theta, \quad \zeta = -\cos\theta, \quad n = 0, 1, 2 \ldots \tag{3}$$

The components of ring density, $g_n(\zeta)$, are represented as a Birnbaum series, Eq. 3, and thus satisfy the Kutta condition at the trailing edge $\gamma(1, \phi) = 0$.

Since the present concern is with the loadings on the ring wing for a specified angle of attack between the wing axis and the free stream velocity and a specified local slope of the wing profile relative to the wing axis, the indirect approach is taken, from which the ring vortex density distribution is solved from the known geometry. Combining the slope of the profile with angle of attack gives an effective geometry or local angle of incidence

$$\alpha(\zeta, \phi) = \sum_{n=0} \alpha_n(\zeta) \cos n\phi \tag{4}$$

$$\alpha_n(\zeta) = \frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta, \quad \zeta = -\cos\theta \tag{5}$$

The ring vortex density distribution, $\gamma(\zeta, \phi)$, must satisfy the boundary condition by removing the radial wash or local angle of incidence, $\alpha(\zeta, \phi)$. For a fixed value of n, the boundary condition in Weissinger's notation is written

$$\alpha_n(\zeta) \cos n\phi = \frac{n\lambda}{4} \int_{-1}^{1} g_n(\zeta') \cos n\phi \, d\zeta' + \frac{1}{2\pi} \int_{-1}^{1} \frac{g_n(\zeta')}{\zeta - \zeta'} \cos n\phi \, d\zeta'$$

$$+ \frac{\lambda}{2\pi} \int_{-1}^{1} g_n(\zeta') \, U_n(\eta) \cos n\phi \, d\zeta' \qquad n = 0, 1, 2 \ldots \tag{6}$$

where

$$\lambda = \frac{L}{2R}, \qquad \eta = \lambda(\zeta - \zeta')$$

The total boundary condition is obtained by summing the boundary conditions for corresponding components of $\alpha(\zeta, \phi)$ and $\gamma(\zeta, \phi)$ over all values of n. Thus

$$\sum_{n=0} \alpha_n(\zeta) \cos n\phi = \sum_{n=0} \frac{n\lambda}{4} \int_{-1}^{1} g_n(\zeta') \cos n\phi \, d\zeta'$$

$$+ \frac{1}{2\pi} \sum_{n=0} \int_{-1}^{1} \frac{g_n(\zeta') \cos n\phi \, d\zeta'}{\zeta - \zeta'}$$

$$+ \frac{\lambda}{2\pi} \sum_{n=0} \int_{-1}^{1} g_n(\zeta') \, U_n(\eta) \cos n\phi \, d\zeta' \tag{7}$$

For a symmetrical ring wing, the local slope of the profile is independent of ϕ. Thus the local slope of the profile is the term of $\alpha(\zeta, \phi)$ on the left side of Eq. 7 for the case n = 0. Setting $\alpha_0(\zeta) = \alpha_0^{**}(\zeta)$ then, for a wing with conicity and camber, the slope of the profile relative to the wing axis can be written

$$\alpha_0^{**}(\zeta) = \iota_0 + \alpha_c^{*}(\zeta) \tag{8}$$

where α_0 is the half-cone angle and $\alpha_0^{*}(\zeta)$ is the slope of the wing profile for the cambered section.

From Fig. 2 it can be seen that for any wing section, the free-stream radial component of velocity is dependent on ϕ and determines the remaining term of $\alpha(\zeta, \phi)$ on the left side of Eq. 7 for the case n = 1. The free-stream radial component of velocity becomes

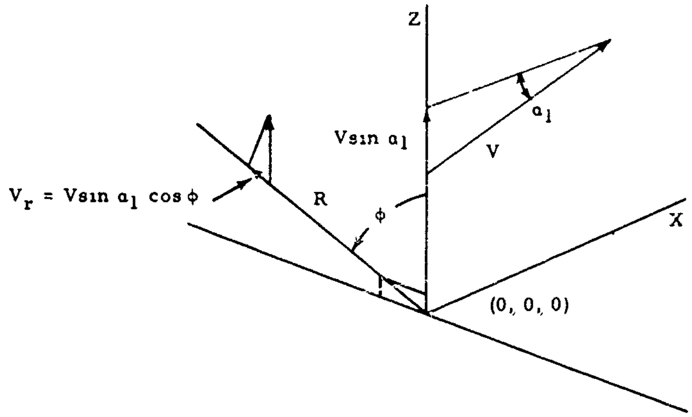

FIG. 2. Free-Stream Angle of Attack as a Function of ϕ.

$$V_r = V \sin \alpha_1 \cos \phi \tag{9}$$

For small α_1, Eq. 9 can be written

$$\frac{V_r}{V} = \alpha_1 \cos \phi, \qquad \alpha_1 \text{ constant} \tag{10}$$

where α_1 is the angle of attack between the wing axis and the free stream in the XZ plane and $\alpha_1 \cos \phi$ is the angle of attack in the XR plane.

Combining the slope of the profile with free-stream angle of attack, Eq. 8 and 10, and substituting in Eq. 7, the total boundary condition becomes

$$\begin{aligned} \alpha_0^{**}(\zeta) + \alpha_1 \cos \phi = {} & \frac{1}{2\pi} \int_{-1}^{1} \frac{g_0^{**}(\zeta')d\zeta'}{\zeta - \zeta'} + \frac{\lambda}{2\pi} \int_{-1}^{1} g_0^{**}(\zeta')\, U_0(\eta) d\zeta' \\ & + \frac{\lambda}{4} \int_{-1}^{1} g_1(\zeta') \cos \phi \, d\zeta' + \frac{1}{2\pi} \int_{-1}^{1} \frac{g_1(\zeta') \cos \phi \, d\zeta'}{\zeta - \zeta'} \\ & + \frac{\lambda}{2\pi} \int_{-1}^{1} g_1(\zeta')\, U_1(\eta) \cos \phi \, d\zeta' \end{aligned} \tag{11}$$

where $g_o^{**}(\zeta) = g_o^{*}(\zeta) + g_o(\zeta)$ and $\alpha_n(\zeta) = g_n(\zeta) = 0, \quad n = 2, 3, \ldots$

Referring to Eq. 3, 5, and 6, the coefficients of $g_1(\zeta)$ and $g_0^{**}(\zeta)$ can easily be solved for in terms of the coefficients of $\alpha_1(\zeta)$ and $\alpha_0^{**}(\zeta)$ from Tables 1 and 2 for the case n = 0, n = 1. With

$$\gamma(\zeta, \phi) = V[g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \tag{12}$$

and using Eq. 1, the net loading per unit area on the wing surface becomes

$$\Delta p = \rho V^2 [g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \tag{13}$$

TABLE 1. Matrix Needed To Solve for $C_{\rho 0}$ in Eq. 28 for n = 0

λ	M_0^{-1}			
0.0	1	0	0	0
	0	-2	0	0
	0	0	-2	0
	0	0	0	-2
0.5	1.080	0	-0.080	0
	0.172	-2.187	-0.013	0.028
	0.027	0	-2.036	0
	-0.001	0.009	0.000	-2.012
1.0	1.202	0	-0.202	0
	0.484	-2.521	-0.082	0.119
	0.118	0	-2.155	0
	-0.002	0.039	0.000	-2.054

TABLE 2. Matrix Needed To Solve for $C_{\rho 1}$ in Eq. 28 for n = 1

λ	M_1^{-1}			
0.0	1	0	0	0
	0	-2	0	0
	0	0	-2	0
	0	0	0	-2
0.5	0.549	0.381	0.068	0.003
	-0.119	-1.851	-0.015	-0.016
	-0.009	-0.006	-1.982	0.000
	0.001	-0.004	0.000	-1.995
1.0	0.398	0.442	0.141	0.021
	-0.208	-1.645	-0.073	-0.076
	-0.034	-0.038	-1.904	-0.002
	0.000	-0.022	0.000	-1.970

Recalling that $a_0^{**}(\zeta)$ is the slope of the wing profile relative to the wing axis, then, from Fig. 1 the normal force per unit area can be written

$$\frac{dY}{dX} = \Delta p \cos a_0^{**}(\zeta) \tag{14}$$

$$\frac{dY}{dX} = \rho V^2 [g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \cos a_0^{**}(\zeta) \tag{15}$$

$$\frac{dY}{d\zeta} = \rho V^2 \frac{L}{2} [g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \cos a_0^{**}(\zeta) \tag{16}$$

where

$$\zeta = \frac{2X}{L}, \qquad \frac{-L}{2} \leq X \leq \frac{L}{2}, \qquad -1 \leq \zeta \leq 1$$

Equation 16 can also be interpreted as the local lift normal to the wing axis per unit length per unit span, and for an element of wing span $Rd\phi$, the local lift in the pitch plane per unit length (Fig. 1) becomes

$$\frac{dY}{d\zeta} \cos\phi \, Rd\phi \tag{17}$$

The total lift normal to the wing axis per unit length is then

$$\int_0^{2\pi} \frac{dY}{d\zeta} \cos\phi \, Rd\phi = \int_0^{2\pi} \rho V^2 \frac{L}{2} [g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \cos a^{**}(\zeta) \cos\phi \, Rd\phi \tag{18}$$

and the total lift normal to the wing axis becomes

$$Y = \int_{-1}^{1} \int_0^{2\pi} \frac{dY}{d\zeta} \cos\phi \, Rd\phi \, d\zeta$$

$$= \frac{\rho V^2 RL}{2} \int_{-1}^{1} \int_0^{2\pi} [g_0^{**}(\zeta) + g_1(\zeta)\cos\phi] \cos a_0^{**}(\zeta) \cos\phi \, d\phi \, d\zeta$$

$$Y = \frac{\rho V^2 RL}{2} \int_{-1}^{1} \int_0^{2\pi} [g_0^{**}(\zeta)\cos\phi \cos a_0^{**}(\zeta) + g_1(\zeta)\cos^2\phi \cos a_0^{**}(\zeta)] \, d\phi \, d\zeta \tag{19}$$

For small values of $\alpha_0^{**}(\zeta)$

$$Y = \frac{\rho V^2 R L}{2} \int_{-1}^{1} \int_{0}^{2\pi} [g_0^{**}(\zeta)\cos\phi + g_1(\zeta)\cos^2\phi]\, d\phi\, d\zeta \tag{20}$$

Since

$$\frac{dM}{d\zeta} = \frac{L\zeta}{2} \frac{dY}{d\zeta}$$

the total moment about the mid-chord point due to forces normal to the wing axis becomes

$$M = \rho V^2 R \left(\frac{L}{2}\right)^2 \int_{-1}^{1} \int_{0}^{2\pi} \zeta\, [g_0^{**}(\zeta)\cos\phi + g_1(\zeta)\cos^2\phi]\, d\phi\, d\zeta \tag{21}$$

Inspection of Eq. 11 and 20 shows that $g_0^{**}(\zeta)$ corresponding to the slope of the profile, $\alpha_0^{**}(\zeta) = \alpha_c + \alpha_0^{*}(\zeta)$, yields a zero net force normal to the wing axis, which is the case for a symmetric ring wing at zero angle of attack. It is apparent then that the total or net normal force is due solely to the density distribution $g_1(\zeta)$ due to angle of attack α_1.

LIFT FORCE ON CYLINDRICAL RING WING

From Eq. 20, the net force normal to the wing axis becomes

$$Y = \frac{\rho V^2 R L}{2} \int_{-1}^{1} \int_{0}^{2\pi} g_1(\zeta)\cos^2\phi\, d\phi\, d\zeta \tag{22}$$

$$Y = \frac{\pi \rho V^2 R L}{2} \int_{-1}^{1} g_1(\zeta)\, d\zeta \tag{23}$$

Substituting Eq. 3 in Eq. 23 for the case n = 1

$$Y = \frac{\pi \rho V^2 R L}{2} \int_{0}^{\pi} \left(C_{01} \operatorname{ctg}\frac{\theta}{2} + \sum_{\nu=1} C_{\nu 1} \sin\nu\theta \right) \sin\theta\, d\theta$$

$$= \pi\rho V^2 \lambda R^2 \left(\int_{0}^{\pi} C_{01} \frac{1+\cos\theta}{\sin\theta} \sin\theta\, d\theta + \int_{0}^{\pi} \sum_{\nu=1} C_{\nu 1} \sin\nu\theta \sin\theta\, d\theta \right) \tag{24}$$

where $\lambda = \dfrac{L}{2R}$.

For $\nu > 1$, the second integral term is zero and Eq. 24 then becomes

$$Y = \pi\rho V^2 \lambda R^2 \left(\pi C_{01} + C_{11} \frac{\pi}{2}\right) \tag{25}$$

By the use of the dimensionless coefficient

$$C_L = \frac{Y}{2\pi R \frac{1}{2}\rho V^2 L} \tag{26}$$

the total normal coefficient of lift due to α_1 becomes

$$C_L = \frac{\pi\lambda R}{L}\left(C_{01} + \tfrac{1}{2} C_{11}\right)$$

$$C_L = \frac{\pi}{2}\left(C_{01} + \tfrac{1}{2} C_{11}\right) \tag{27}$$

Recalling the Fourier and Birnbaum expansions

$$\alpha_n(\zeta) = \frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta, \qquad \zeta = -\cos\theta$$

$$g_n(\zeta) = C_{0n} \operatorname{ctg}\frac{\theta}{2} + \sum_{\nu=1} C_{\nu n} \sin \nu\theta$$

the solution to the boundary condition represented by Eq. 6 can be reduced to the solution of the linear equations (see Appendix)

$$a_{0n} = C_{0n}\left[1 + \frac{\pi n\lambda}{2} + \frac{\lambda}{2\pi}\left(b_{10} + b_{00}\right)\right] + C_{1n}\left[\frac{\pi n\lambda}{4} + \frac{\lambda}{4\pi}\left(b_{00} - b_{20}\right)\right]$$

$$+ \frac{\lambda}{4\pi}\sum_{\nu=2} C_{\nu n}\left(b_{\nu-1,0} - b_{\nu+1,0}\right), \qquad \rho = 0$$

$$a_{\rho n} = C_{\rho n}\left[-\tfrac{1}{2} + \frac{\lambda}{4\pi}\left(b_{\rho-1,\rho} - b_{\rho+1,\rho}\right)\right] + C_{0n}\frac{\lambda}{2\pi}\left(b_{1\rho} + b_{0\rho}\right)$$

$$+ \frac{\lambda}{4\pi}\sum_{\substack{\nu=1 \\ \nu\neq\rho}} C_{\nu n}\left(b_{\nu-1,\rho} - b_{\nu+1,\rho}\right), \qquad \rho = 1, 2 \ldots \tag{28}$$

$$b_{\nu\mu} = \frac{2}{\pi}\int_0^\pi f_\nu(\theta)\cos\mu\,\theta d\theta, \qquad \eta = \lambda(\zeta - \zeta'), \qquad \zeta = -\cos\theta \tag{29}$$

$$f_\nu(\theta) = \int_0^\pi \cos\,\nu\theta'\, U_n(\eta)d\theta' \tag{30}$$

For a cylindrical wing, $\alpha_0^{**}(\zeta) = 0$, at angle of attack α_1 the local angle of incidence, Eq. 5, becomes

$$\alpha_1 = \frac{a_{01}}{2} + \sum_{\rho=1} a_{\rho 1}\cos\rho\theta, \qquad \rho = 1, 2 \ldots \tag{31}$$

$$a_{\rho 1} = \frac{2}{\pi}\int_0^\pi \alpha_1 \cos\,\rho\theta d\theta \tag{32}$$

From Eq. 32, the Fourier coefficients for α_1, constant, are then

$$a_{\rho 1} = 0, \qquad \rho = 1, 2 \ldots \tag{33}$$
$$a_{01} = 2\alpha_1$$

In matrix notation Eq. 28 becomes

$$\begin{bmatrix} a_{0n} \\ a_{1n} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_{\rho n} \end{bmatrix} = M_n \begin{bmatrix} C_{0n} \\ C_{1n} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_{\rho n} \end{bmatrix}$$

where the elements of M_n are calculated from Eq. 28 with elements of the inverse M_n^{-1} shown in Tables 1 and 2 for n = 0, n = 1.

Setting n = 1 in the matrix equation and multiplying both sides by M_1^{-1}, the solution for the coefficients $C_{\rho 1}$, $\rho = 0, 1, 2 \ldots$ takes the form

$$\begin{bmatrix} C_{01} \\ C_{11} \\ \cdot \\ \cdot \\ \cdot \\ C_{\rho 1} \end{bmatrix} = M_1^{-1} \begin{bmatrix} a_{01} \\ a_{11} \\ \cdot \\ \cdot \\ \cdot \\ a_{\rho 1} \end{bmatrix}$$

From Eq. 33

$$\begin{bmatrix} C_{01} \\ C_{11} \\ \cdot \\ \cdot \\ \cdot \\ C_{\rho 1} \end{bmatrix} = M_1^{-1} \begin{bmatrix} 2\alpha_1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{34}$$

Inspection of Eq. 27 and setting $\alpha_1 = 1$ in Eq. 34 show that the normal coefficient of lift of a cylindrical ring wing per unit angle of attack, α_1, can be written

$$C_L = \frac{\pi}{2}\left[C_{01}^{(0)} + \tfrac{1}{2}\, C_{11}^{(0)}\right] \tag{35}$$

$$\begin{bmatrix} C_{01}^{(0)} \\ C_{11}^{(0)} \\ C_{21}^{(0)} \\ \cdot \\ \cdot \\ \cdot \\ C_{\rho 1}^{(0)} \end{bmatrix} = M_1^{-1} \begin{bmatrix} 2 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{36}$$

where M_1^{-1} is determined by the value of λ.

Taking $\lambda = 1$ and recalling a rule for matrix multiplication, the values in the first column of Table 2 multiplied by the factor 2 in Eq. 36 gives a solution to the coefficients $C_{\rho 1}^{(0)}$, $\rho = 0, 1, 2, \ldots$. Thus for $\lambda = 1$, $C_{01}^{(0)} = 0.796$, $C_{11}^{(0)} = -0.415$, and for a unit angle of attack in radians, $C_L = 0.92$. The solution to Eq. 36 can also be found in Table 3. Figure 3 shows the trend of the slope of the lift curve in degrees for different values of λ.

For the case of a ring wing with $\lambda = 0$, $C_{01}^{(0)} = 2$, $C_{11}^{(0)} = 0$ and from Eq. 35 for unit angle of attack in radians, $C_L = \pi$.

TABLE 3. The Birnbaum Coefficients $C_{\rho 1}^{(0)}$ of the Vortex Density g_1 for Unit Angle of Attack

λ	$C_{01}^{(0)}$	$C_{11}^{(0)}$	$C_{21}^{(0)}$	$C_{31}^{(0)}$	$C_{41}^{(0)}$
0	2.000	0	0	0	0
0.5	1.098	-0.238	-0.018	0.002	0
1	0.796	-0.415	-0.068	0.000	0

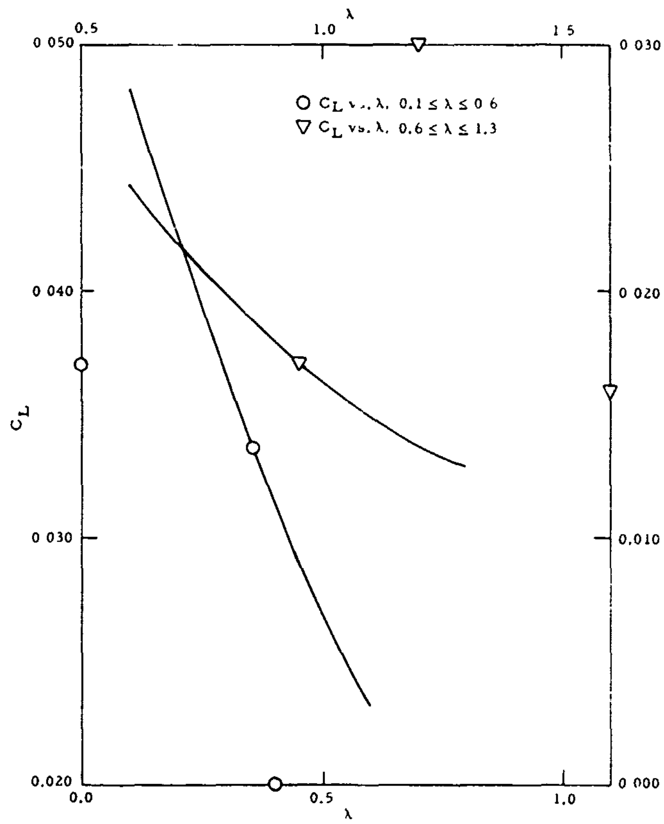

FIG. 3. Total Lift Coefficient per Unit Angle of Attack, deg, versus λ.

MOMENT FOR CYLINDRICAL RING WING

From Eq. 21, the total moment about the mid-chord point is written

$$M = \rho V^2 R \left(\frac{L}{2}\right)^2 \int_{-1}^{1} \int_{0}^{2\pi} \zeta g_1(\zeta) \cos^2\phi \; d\phi \; d\zeta \tag{37}$$

where from Fig. 1 the moment is negative clockwise.

Integration on ϕ gives

$$M = \pi \rho V^2 \lambda^2 R^3 \int_{-1}^{1} \zeta g_1(\zeta) d\zeta \tag{38}$$

Using the Birnbaum series expansion for $g_1(\zeta)$ in Eq. 3 and, recalling that $\zeta = -\cos\theta$, Eq. 38 becomes

$$M = \pi \rho V^2 \lambda^2 R^3 \int_0^{\pi} - \left(C_{01} \operatorname{ctg} \frac{\theta}{2} + \sum_{\nu=1} C_{\nu 1} \sin \nu\theta \right) \sin\theta \cos\theta \, d\theta$$

$$= \pi \rho V^2 \lambda^2 R^3 \int_0^{\pi} - \left[C_{01} \left(\frac{1+\cos\theta}{\sin\theta} \right) \sin\theta \cos\theta + \sum_{\nu=1} C_{\nu 1} \sin \nu\theta \sin\theta \cos\theta \right] d\theta$$

$$= \pi \rho V^2 \lambda^2 R^3 \int_0^{\pi} - \left[C_{01} (\cos\theta + \cos^2\theta) + \sum_{\nu=1} C_{\nu 1} \sin \nu\theta \frac{\sin 2\theta}{2} \right] d\theta$$

$$= \pi \rho V^2 \lambda^2 R^3 \int_0^{\pi} \left(- C_{01} \cos\theta - C_{01} \cos^2\theta - C_{21} \frac{\sin^2 2\theta}{2} \right) d\theta \tag{39}$$

$$M = \pi \rho V^2 \lambda^2 R^3 \left(-C_{01} \frac{\pi}{2} - C_{21} \frac{\pi}{4} \right) \tag{40}$$

Defining the dimensionless coefficient as

$$C_M = \frac{M}{2\pi R L^2 \frac{1}{2} \rho V^2} \qquad \text{with } C_M \text{ negative clockwise}$$

then from Eq. 40 the total moment coefficient due to normal forces can be written

$$C_M = -\frac{\pi}{16} \left(2C_{01} + C_{21} \right) \tag{41}$$

The moment coefficient for unit angle of attack becomes

$$C_M = -\frac{\pi}{16}\left(2C_{01}^{(0)} + C_{21}^{(0)}\right)$$

and is plotted in Fig. 4 for various values of λ.

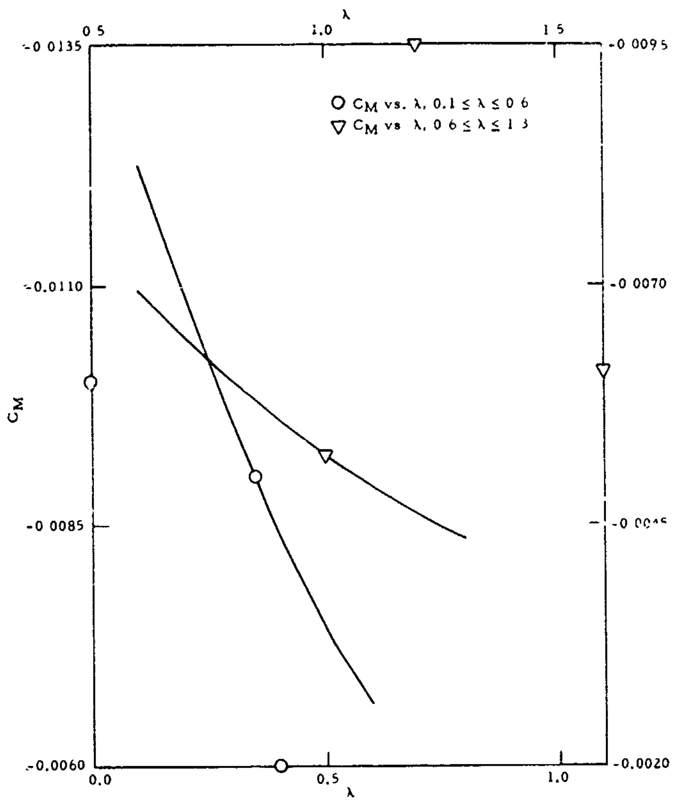

FIG. 4. Total Moment Coefficient per Unit Angle of Attack, deg, Versus λ.

AERODYNAMIC CENTER OF LIFT

Defining the aerodynamic center measured from the mid-chord point as

$$X_{ac} = \frac{M}{Y}$$

or

$$\frac{X_{ac}}{L} = \frac{C_M}{C_L}$$

then from Eq. 27 and 41, the aerodynamic center of lift for ring wings becomes

$$\frac{X_{ac}}{L} = \frac{-\frac{\pi}{16}\left(2C_{01} + C_{21}\right)}{\frac{\pi}{2}\left(C_{01} + \frac{1}{2}C_{11}\right)}$$

$$\frac{X_{ac}}{L} = -\frac{1}{8}\frac{\left(2C_{01} + C_{21}\right)}{\left(C_{01} + \frac{1}{2}C_{11}\right)} \tag{42}$$

For $\lambda = 0$, and from Table 2, together with Eq. 34, $C_{01} = 2a_1$, $C_{11} = C_{21} = 0$. Equation 42 then gives the well-known two-dimensional result

$$X_{ac} = -\frac{L}{4}, \qquad -\frac{L}{2} \leq X \leq \frac{L}{2} \tag{43}$$

For $\lambda = 1$, from Table 2, Eq. 34 and 42,

$$C_{01} = 0.398\ (2a_1)$$

$$C_{11} = -0.208\ (2a_1)$$

$$C_{21} = -0.034\ (2a_1)$$

$$\frac{X_{ac}}{L} = -\frac{1}{8}\,\frac{(1.592-0.068)a_1}{(0.796-0.208)a_1}$$

$$X_{ac} = -0.325L \tag{44}$$

The dependency of the wing aerodynamic center on the shape parameter, λ, of the ring wing is shown in Fig. 5. Relative to a two-dimensional airfoil of unit-span length, a chord-diameter ratio of $\lambda = 1$ for a cylindrical wing of span length $2\pi R$ will cause a 30% shift in the aerodynamic center toward the leading edge.

CONCLUSIONS

A ring wing with a small profile slope (Fig. 1) is replaced by a continuous distribution of vortex rings of constant radius. Within the limits of linearized theory, the velocities and pressures on the cambered ring wing with conicity are represented by the velocities and pressures on a cylindrical vortex sheet. The boundary condition on the ring wing is given in Eq. 11. Inspection of Eq. 11 and 20 shows that

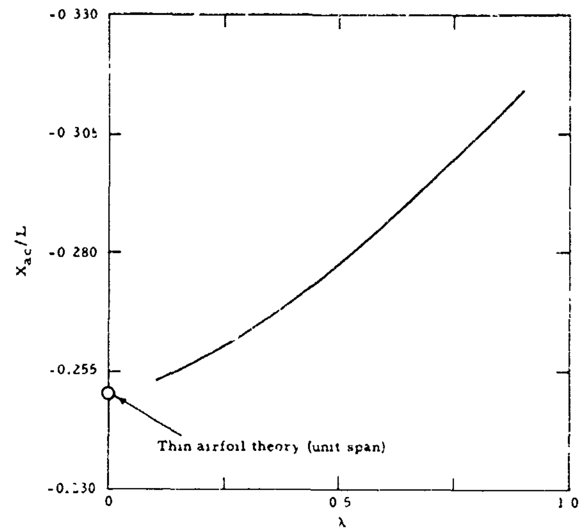

FIG. 5. Aerodynamic Center Versus λ.

the slope of a symmetric ring wing profile, $\alpha_0^{**}(\zeta)$, gives a zero net force normal to the wing axis and that the total normal force results from free-stream angle of attack. Thus as a first order approximation the normal force on a ring wing of arbitrary profile shape is equivalent to the normal force on a cylindrical ring wing.

For a cylindrical wing at angle of attack, α_1, where $\alpha^{**}(\zeta) = g^{**}(\zeta) = 0$, the boundary condition is defined in Eq. 28 for n = 1, i. e., the remaining terms containing α_1 and g_1 in Eq. 11. A reduction of the boundary condition to the linear algebraic form in Eq. 28 is given in the Appendix. For n = 1, with coefficients of radial wash, $a_{\rho 1}$, given, the coefficients of the Birnbaum expansion for vorticity, $C_{\rho 1}$, can be determined from the set of linear algebraic equations shown in Eq. 28. This gives the lift and moment for a cylindrical wing as defined in Eq. 27 and 41.

Figure 6 gives the total lift coefficient, C_L, versus α_1 in degrees for different values of λ. The lift and moment coefficients per unit angle of attack in degrees are plotted for different values of the ring wing parameter λ as shown in Fig. 3 and 4. The location of the wing aerodynamic center measured from the mid-chord point is shown in Fig. 5 for different values of λ. For λ = 0, the aerodynamic center is located at the 1/4-chord point, which is the two-dimensional result. An example of the three-dimensional effect due to curvature of the chord plane is demonstrated for the case λ = 1 which indicates about a 30% shift in the aerodynamic center upstream from the 1/4-chord point.

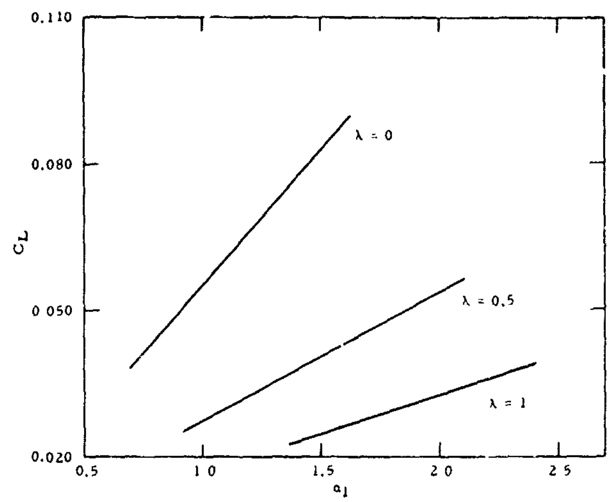

FIG. 6. Total Lift Coefficient Versus a_1, deg, for Various Values of λ.

Tables 1 and 2, obtained from Eq. 28, give the Birnbaum coefficients required to calculate the lift, moment, and aerodynamic center for isolated cylindrical wings. Table 3 was obtained from Eq. 36 and Table 2. Recent investigations show a significant change in aerodynamic characteristics of the ring wing in the presence of a central body. It has been found that because of the presence of axial forces further changes are apparent in wing loadings and location of the aerodynamic center of lift due to wing camber and conicity. The results of this study will be published in the near future.

Appendix

SOLUTION TO THE BOUNDARY CONDITION

The general boundary condition is written

$$a_n(\zeta) = \frac{n\lambda}{4}\int_{-1}^{1} g_n(\zeta')d\zeta' + \frac{1}{2\pi}\int_{-1}^{1}\frac{g_n(\zeta')}{\zeta - \zeta'}\,d\zeta' + \frac{\lambda}{2\pi}\int_{-1}^{1} g_n(\zeta')U_n(\eta)d\zeta' \qquad \text{(A-1)}$$

From Eq. 3 the second term in Eq. A-1 becomes

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{g_n(\zeta')d\zeta'}{\zeta - \zeta'} = \frac{C_{0n}}{2} - \frac{1}{2}\sum_{\nu=1} C_{\nu n}\cos\nu\theta \qquad n = 0, 1, 2 \ldots \qquad \text{(A-2)}$$

and the integral expression in the first term of Eq. A-1 becomes

$$\int_{-1}^{1} g_n(\zeta')d\zeta' = \pi C_{0n} + \frac{\pi}{2}\,C_{1n} \qquad \text{(A-3)}$$

The last term in Eq. A-1 is then

$$\int_{-1}^{1} g_n(\zeta')U_n(\eta)d\zeta' = \int_{0}^{\pi}\left(C_{0n}\operatorname{ctg}\frac{\theta'}{2} + \sum_{\nu=1} C_{\nu n}\sin\nu\theta'\right)U_n(\eta)\sin\theta'\,d\theta'$$

$$= \int_{0}^{\pi}\left(C_{0n} + C_{0n}\cos\theta' + \sum_{\nu=1} C_{\nu n}\sin\nu\theta'\sin\theta'\right)U_n(\eta)d\theta'$$

$$= \int_{0}^{\pi} C_{0n}U_n(\eta)d\theta' + \int_{0}^{\pi} C_{0n}\cos\theta'\,U_n(\eta)d\theta'$$

$$+ \int_{0}^{\pi}\sum_{\nu=1} C_{\nu n}\sin\nu\theta'\sin\theta'\,U_n(\eta)d\theta' \qquad \text{(A-4)}$$

From the identity

$$2\sin\nu\theta'\sin\theta' = -\cos(\nu + 1)\theta' + \cos(\nu - 1)\theta' \qquad \text{(A-5)}$$

the last integral term in Eq. A-4 becomes

$$\int_0^{\pi} \sum_{\nu=1} C_{\nu n} \sin \nu\theta' \sin \theta' U_n(\eta) d\theta'$$

$$= \sum_{\nu=1} C_{\nu n} \int_0^{\pi} \left[\frac{\cos(\nu - 1)\theta' - \cos(\nu + 1)\theta'}{2} \right] U_n(\eta) d\theta' \tag{A-6}$$

$$f_\nu(\theta) = \int_0^{\pi} \cos \nu\theta' \, U_n(\eta) d\theta' \tag{A-7}$$

Expanding $f_\nu(\theta)$ in an even Fourier series

$$f_\nu(\theta) = \frac{b_{\nu 0}}{2} + \sum_{\mu=1} b_{\nu\mu} \cos \mu\theta \tag{A-8}$$

$$b_{\nu\mu} = \frac{2}{\pi} \int_0^{\pi} f_\nu(\theta) \cos \mu\theta \, d\theta$$

$$f_0(\theta) = \int_0^{\pi} U_n(\eta) d\theta'$$

$$f_1(\theta) = \int_0^{\pi} \cos \theta' U_n(\eta) d\theta'$$

$$f_{\nu-1}(\theta) = \int_0^{\pi} \cos (\nu - 1)\theta' \, U_n(\eta) d\theta'$$

$$f_{\nu+1}(\theta) = \int_0^{\pi} \cos (\nu + 1)\theta' U_n(\eta) d\theta' \tag{A-9}$$

From Eq. A-9, Eq. A-6 becomes

$$\int_0^{\pi} \sum_{\nu=1} C_{\nu n} \sin \nu\theta' \sin \theta' U_n(\eta) d\theta' = \sum_{\nu=1} C_{\nu n} \frac{[f_{\nu-1}(\theta) - f_{\nu+1}(\theta)]}{2} \tag{A-10}$$

Equation A-4 is then

$$\int_{-1}^{1} g_n(\zeta')U_n(\eta)d\zeta' = C_{0n}[f_0(\theta)+f_1(\theta)] + \sum_{\nu=1} C_{\nu n}\frac{[f_{\nu-1}(\theta) - f_{\nu+1}(\theta)]}{2} \quad \text{(A-11)}$$

$$a_n(\zeta) = \frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta \quad \text{(A-12)}$$

From Eq. A-7 and A-9

$$b_{\nu\mu} = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} \cos \nu\theta' U_n(\eta)d\theta' \cos \mu\theta \, d\theta$$

$$b_{\nu\mu} = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} \cos \nu\theta' \cos \mu\theta U_n(\eta)d\theta' \, d\theta \quad \text{(A-13)}$$

From Eq. A-2, A-3, A-11, and A-12, Eq. A-1 becomes

$$\frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta = \frac{n\lambda}{4}\left(\pi C_{0n} + \frac{\pi}{2} C_{1n}\right)$$
$$+\left(\frac{C_{0n}}{2} - \frac{1}{2}\sum_{\nu=1} C_{\nu n} \cos \nu\theta\right) + \frac{\lambda C_{0n}}{2\pi}[f_0(\theta) + f_1(\theta)]$$
$$+\frac{\lambda}{2\pi}\sum_{\nu=1} C_{\nu n}\left[\frac{f_{\nu-1}(\theta) - f_{\nu+1}(\theta)}{2}\right] \quad n = 0,1,2 \ldots \quad \text{(A-14)}$$

From Eq. A-9. Eq. A-14 is then

$$\frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta = \frac{n\lambda}{4}\left(\pi C_{0n} + \frac{\pi}{2} C_{1n}\right) + \left(\frac{C_{0n}}{2} - \frac{1}{2}\sum_{\nu=1} C_{\nu n} \cos \nu\theta\right)$$
$$+\frac{\lambda C_{0n}}{2\pi}\left(\frac{b_{00}}{2} + \sum_{\mu=1} b_{0\mu} \cos \mu\theta + \frac{b_{10}}{2} + \sum_{\mu=1} b_{1\mu} \cos \mu\theta\right)$$
$$+\frac{\lambda}{2\pi}\sum_{\nu=1}\frac{C_{\nu n}}{2}\left[\left(\frac{b_{\nu-1,0}}{2} + \sum_{\mu=1} b_{\nu-1,\mu} \cos \mu\theta\right)\right.$$
$$\left.-\left(\frac{b_{\nu+1,0}}{2} + \sum_{\mu=1} b_{\nu+1,\mu} \cos \mu\theta\right)\right] \quad \text{(A-15)}$$

Equation A-15 can be written

$$\frac{a_{0n}}{2} + \sum_{\rho=1} a_{\rho n} \cos \rho\theta = \frac{n\lambda}{4}\left(\pi C_{0n} + \frac{\pi}{2} C_{1n}\right) + \left(\frac{C_{0n}}{2} - \frac{1}{2}\sum_{\nu=1} C_{\nu n} \cos \nu\theta\right)$$

$$+ \frac{\lambda C_{0n}}{2\pi}\left(\frac{b_{00}}{2} + \frac{b_{10}}{2}\right) + \frac{\lambda}{2\pi}\sum_{\nu=1}\frac{C_{\nu n}}{2}\left(\frac{b_{\nu-1,0}}{2} - \frac{b_{\nu+1,0}}{2}\right)$$

$$+ \frac{\lambda}{2\pi} C_{0n} \sum_{\mu=1} (b_{0\mu} + b_{1\mu}) \cos \mu\theta$$

$$+ \frac{\lambda}{2\pi}\sum_{\nu=1}\frac{C_{\nu n}}{2}\sum_{\mu=1}(b_{\nu-1,\mu} - b_{\nu+1,\mu}) \cos \mu\theta \qquad \text{(A-16)}$$

Equation A-16 represents the general boundary condition for ring wings in series form. Let

$$\sum_{\nu=1} C_{\nu n} \cos \nu\theta = \sum_{\rho=1} C_{\rho n} \cos \rho\theta$$

and let $\rho = \mu$ in Eq. A-16.

Then

$$a_{0n} + \sum_{\rho=1} 2a_{\rho n} \cos \rho\theta = \frac{n\lambda\pi}{2} C_{0n} + \frac{n\lambda\pi}{4} C_{1n} + C_{0n} - \sum_{\rho=1} C_{\rho n} \cos \rho\theta$$

$$+ \frac{\lambda}{2\pi} C_{0n}(b_{00} + b_{10}) + \frac{\lambda}{4\pi}\sum_{\nu=1} C_{\nu n}(b_{\nu-1,0} - b_{\nu+1,0})$$

$$+ \frac{\lambda}{\pi} C_{0n} \sum_{\rho=1}(b_{0\rho} + b_{1\rho}) \cos \rho\theta$$

$$+ \frac{\lambda}{2\pi}\sum_{\nu=1} C_{\nu n}\sum_{\rho=1}(b_{\nu-1,\rho} - b_{\nu+1,\rho}) \cos \rho\theta \qquad \text{(A-17)}$$

A solution to the general wing boundary condition can now be represented by equating terms in Eq. A-17 containing $\cos \rho\theta$, $\rho = 0, 1, 2 \ldots$

For the case $\rho = 0$, or terms independent of $\cos \rho\theta$

$$a_{0n} = C_{0n}\left[1 + \frac{\pi n\lambda}{2} + \frac{\lambda}{2\pi}(b_{10} + b_{00})\right] + C_{1n}\left[\frac{\pi n\lambda}{4} + \frac{\lambda}{4\pi}(b_{00} - b_{20})\right]$$

$$+ \frac{\lambda}{4\pi}\sum_{\nu=2} C_{\nu n}(b_{\nu-1,0} - b_{\nu+1,0}), \quad \rho = 0 \tag{A-18}$$

For $\rho \neq 0$ or terms dependent on $\cos\rho\theta$

$$\sum_{\rho=1} 2a_{\rho n}\cos\rho\theta = -\sum_{\rho=1} C_{\rho n}\cos\rho\theta + \frac{\lambda}{\pi} C_{0n}\sum_{\rho=1}(b_{0\rho} + b_{1\rho})\cos\rho\theta$$

$$+ \frac{\lambda}{\pi}\sum_{\nu=1}\frac{C_{\nu n}}{2}\sum_{\rho=1}(b_{\nu-1,\rho} - b_{\nu+1,\rho})\cos\rho\theta \tag{A-19}$$

Isolating the term for $\nu = \rho$, Eq. A-19 becomes

$$\sum_{\rho=1} 2a_{\rho n}\cos\rho\theta = -\sum_{\rho=1} C_{\rho n}\cos\rho\theta + \frac{\lambda C_{0n}}{\pi}\sum_{\rho=1}(b_{0\rho} + b_{1\rho})\cos\rho\theta$$

$$+ \frac{\lambda}{\pi}\frac{C_{\rho n}}{2}(b_{\rho-1,\rho} - b_{\rho+1,\rho})_{\nu=\rho}\cos\rho\theta$$

$$+ \frac{\lambda}{\pi}\sum_{\substack{\nu=1\\ \nu\neq\rho}}\frac{C_{\nu n}}{2}\sum_{\rho=1}(b_{\nu-1,\rho} - b_{\nu+1,\rho})\cos\rho\theta \tag{A-20}$$

Expanding Eq. A-20, equating corresponding terms and dividing through by $\cos\rho\theta$, $\rho = 1, 2, \ldots$

$$a_{\rho n} = -\frac{C_{\rho n}}{2} + \frac{\lambda C_{0n}}{2\pi}(b_{0\rho} + b_{1\rho}) + \frac{\lambda}{4\pi} C_{\rho n}(b_{\rho-1,\rho} - b_{\rho+1,\rho})$$

$$+ \frac{\lambda}{4\pi}\sum_{\substack{\nu=1\\ \nu\neq\rho}} C_{\nu n}(b_{\nu-1,\rho} - b_{\nu+1,\rho}) \tag{A-21}$$

$$a_{\rho n} = C_{\rho n}\left[-\frac{1}{2} + \frac{\lambda}{4\pi}(b_{\rho-1,\rho} - b_{\rho+1,\rho})\right] + C_{0n}\frac{\lambda}{2\pi}(b_{1\rho} + b_{0\rho})$$

$$+ \frac{\lambda}{4\pi}\sum_{\substack{\nu=1\\ \nu\neq\rho}} C_{\nu n}(b_{\nu-1,\rho} - b_{\nu+1,\rho}), \quad \rho = 1, 2 \ldots \tag{A-22}$$

where the coefficients $b_{\nu\rho}$, $\rho = \mu$, are defined by Eq. A-7 and A-9.

Equations A-18 and A-22 represent a set of linear algebraic equations with unknown coefficients, $C_{\rho n}$, the solutions of which satisfy the ring wing boundary condition in Eq. A-1. A solution of $C_{\rho n}$ is available on an IBM 7090 computer for any value of λ and any component of local incidence angle, $\alpha_n(\zeta)$ represented by the set $a_{\rho n}$, $\rho = 0, 1, 2, \ldots$.

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